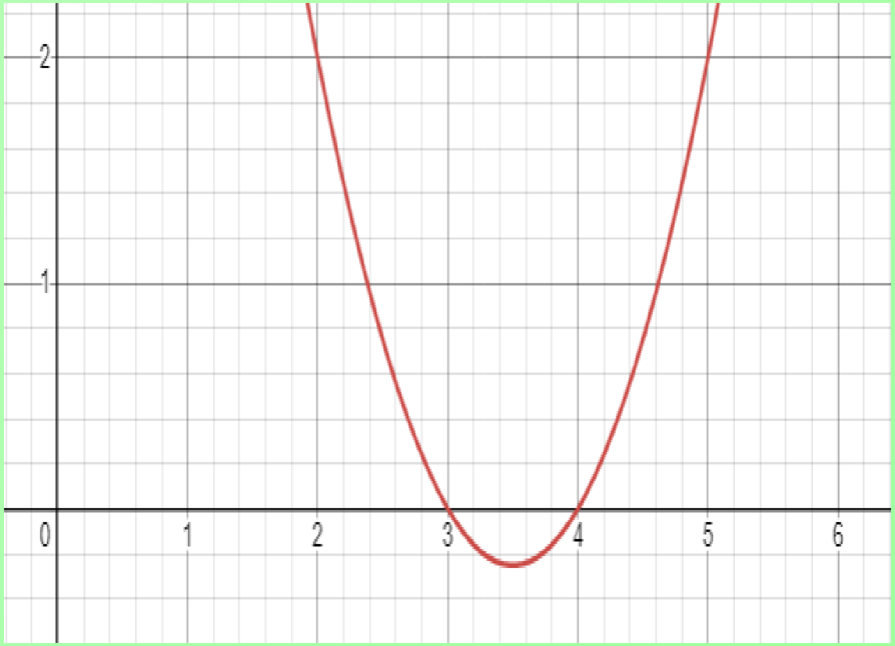
Tuesday, May 27, 2025

1.1 Argument & Proof

er Question – give your answer in set not

Find the set of values of x for which $(x - 1)(x - 4)$ ¿ $2(x - 4)$.



A Proof

A1	Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion. Disproof by counter example.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- set out a clear proof with the correct use of symbols, such as $=$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$, $\equiv$, $\therefore$, $\because$
- understand that considering examples can be useful in looking for structure, but this does not constitute a proof.

A Proof

A1

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion.

Disproof by counter example.

Notes

- At A-level 25% (20% at AS) of the assessment material must come from Assessment Objective 2 (reason, interpret and communicate mathematically). A focus on clear reasoning, mathematical argument and proof using precise mathematical language and notation should underpin the teaching of this specification. Students should become familiar with the mathematical notation found in Appendix A of the specification.
- It will not be essential to use any particular notation when writing answers to exam questions, but some questions could assess understanding of this notation.
- Students should understand the sets of numbers $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

1.1 Argument & Proof

Logical Consequence

Use of particular arrows to show whether one thing **implies** another and perhaps vice versa.

means: ‘A implies B’

‘if A is true then B is true’

‘A is sufficient for B’

Example 1

A: is a prime number > $\Rightarrow$ 2 B: is an odd number

1.1 Argument & Proof

Logical Consequence

Use of particular arrows to show whether one thing **implies** another and perhaps vice versa.

means: ‘A is implied by B’

‘if B is true then A is true’

‘A is necessary for B’

Example 2

A: a shape has 4 sides $\Leftarrow$ B: the shape is a square

1.1 Argument & Proof

Logical Equivalence

If implies **AND** implies then we have a logical **equivalence**. This is written:

We can say: P is equivalent to Q

Q is true if and only if P is true

Example 3

A: is odd ⇔ B: is odd

1.1 Argument & Proof

You try…

Place the correct logical operator between these statements:

$\Rightarrow$

The object is a cube. The object has six faces.

$\Rightarrow$

$\Leftarrow$

$\Leftrightarrow$

is a prime number. has exactly two factors.

1.1 Argument & Proof

Example 4

Insert the correct symbol:

$\Rightarrow$

$\square$

$\Leftarrow$

$\square$

Complete the statement:

$$x = \pm 4$$

1.1 Argument & Proof

Necessary & Sufficient – In Summary:

A is **sufficient** but not **necessary** for B

A is **necessary** but not **sufficient** for B

A is **necessary and sufficient** for B

Complete the table & Q1 of ex1.1

Other Notation

means is in the set of all real numbers

means is in the set of all natural numbers

means is in the set of all rational numbers

means is in the set of all integers

means the empty set.

1.1 Argument & Proof

A method of proof that can be used to prove that a conjecture is true is ***deduction***.

This involves a logical argument as to why the conjecture must be true. This will often require you to use algebra.

This is also known as ***direct proof***.

1.1 Argument & Proof

Things you need to know for proof by deduction:

- All even numbers can be written in the form
- All odd numbers can be written in the form or
- Consecutive numbers: , , …
- Consecutive even numbers: , , …
- Consecutive odd numbers: , , …
- Any two *unrelated* even numbers: ,
- Any two *unrelated* odd numbers: ,
- A rational number can be expressed as where and are integers and

1.1 Argument & Proof

Example 5

Conjecture: The sum of any two consecutive odd numbers is a multiple of 4.

Let the consecutive odd numbers be and

which is a multiple of 4

Hence, the sum of any two consecutive odd numbers is a multiple of 4

1.1 Argument & Proof

Example 6

Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

We need a slightly different approach here. We are not told whether n is odd or even so we need to consider both cases.

Case 1: n is even

Let

which is even

Case 2: n is odd

Let

which is even is never odd.

1.1 Argument & Proof

Another method of proof is called ***proof by exhaustion.***

In this method, we have to check all possibilities. This can only be done if there is a small number of options, or the options can be split up into different cases, i.e. odd cases, even cases.

All cases must be true for proof by exhaustion to work.

A single counter-example would disprove the result.

1.1 Argument & Proof

Example 7

Prove, by exhaustion, that is not divisible by 3, where is an integer and .

p	***p^2 + 1***	***Divisible by 3?***
6	*37*	*No*
7	*50*	*No*
8	*65*	*No*
9	*82*	*No*
10	*101*	*No*

is not divisible by 3 for integer values of such that

1.1 Argument & Proof

Example 8

Prove that no square number ends in an 8

We cannot try ALL numbers in existence so consider numbers ending in 1, when we square them, it also ends in 1. Extend this to numbers ending in 2, 3

...1	...1
...2	...4
...3	...9
...4	...6
...5	...5
...6	...6
...7	...9
...8	...4
...9	...1
...0	...0

Consider numbers ending in 0-9. Their squares end with 0, 1, 4, 5, 6 or 9.

Hence no square number ends in an 8.

1.1 Argument & Proof

Proof by counter-example is used to prove that a conjecture is not true.

A counter-example is a single example where the conjecture does not work and so disproves the conjecture altogether.

1.1 Argument & Proof

Example 9

Prove by counter-example, that the statement "is prime for all integers " is false.

Let

21 has factors 1, 3, 7 and 21, so is not prime.

This disproves the statement that 'is prime for all integers '

1.1 Argument & Proof

Example 10

Disprove by counter example: If then

4 2 Yes
4 -2 No

If then but

CGP Exercise 1.1